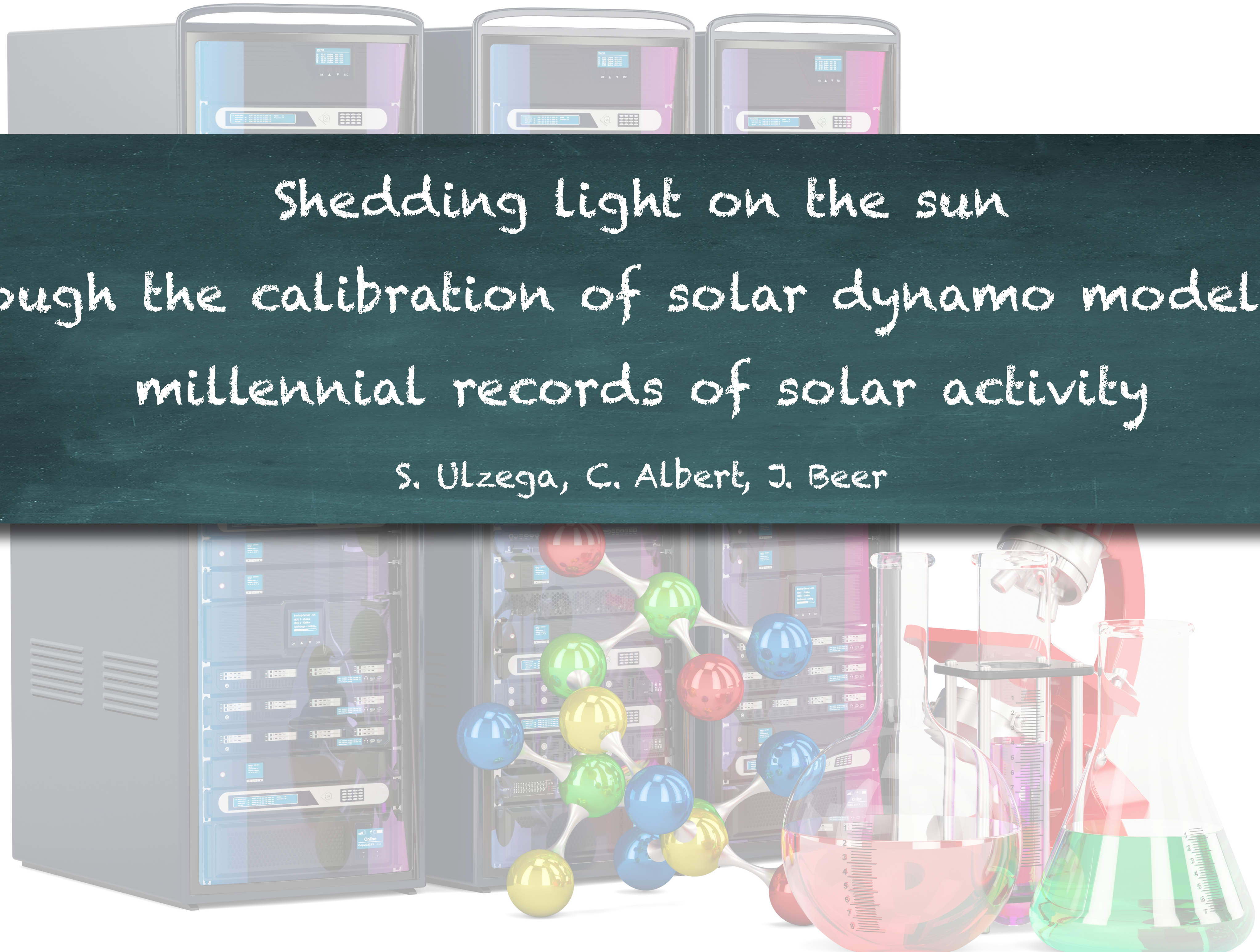
Zurich University of Applied Sciences

Institute of Computational Life Sciences



Shedding light on the sun through the calibration of solar dynamo models on millennial records of solar activity

S. Ulzega, C. Albert, J. Beer

SCOSTEP, 16.05.2023

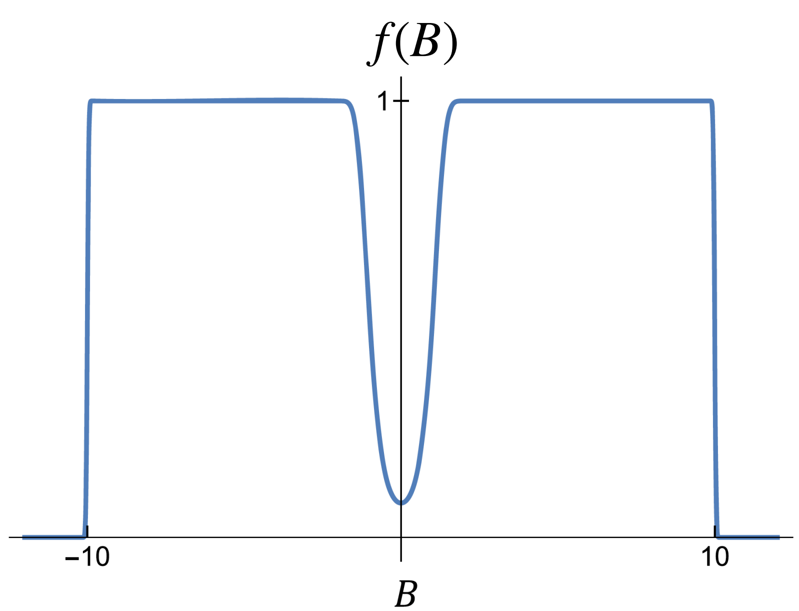
The solar dynamo: a delay ODE model

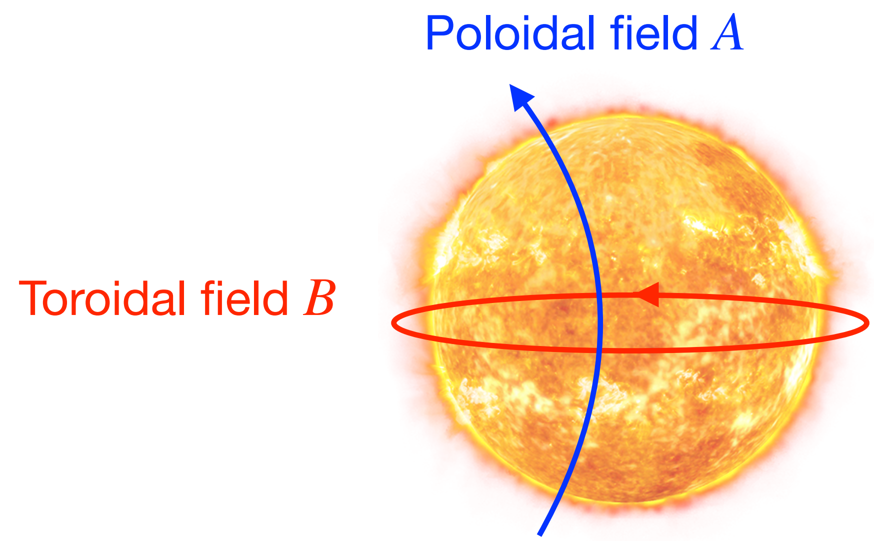
- The **MHD dynamo** mechanism is widely accepted to be the cause of stellar magnetism

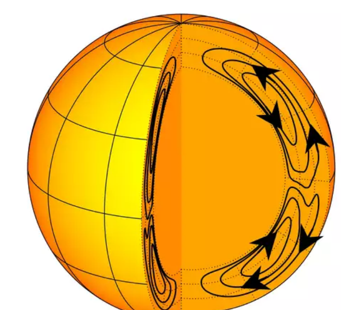
$$\frac{dB(t)}{dt} = \frac{\Omega_0}{L} A(t - T_0) - \frac{B(t)}{\tau}$$

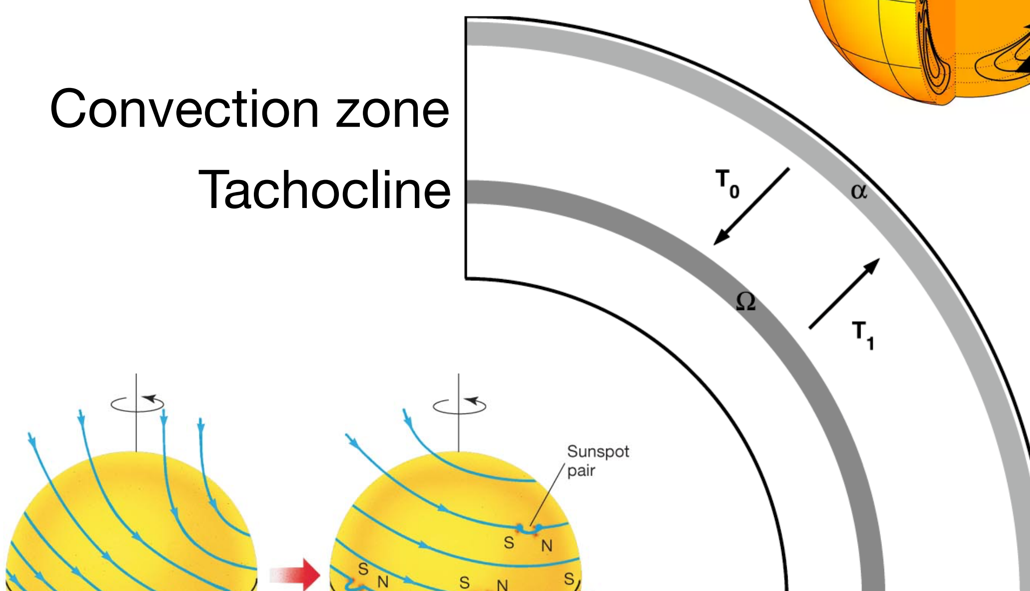
$$\frac{dA(t)}{dt} = \alpha_0 \cdot f\left(B(t - T_1)\right) \cdot B(t - T_1) - \frac{A(t)}{\tau}$$

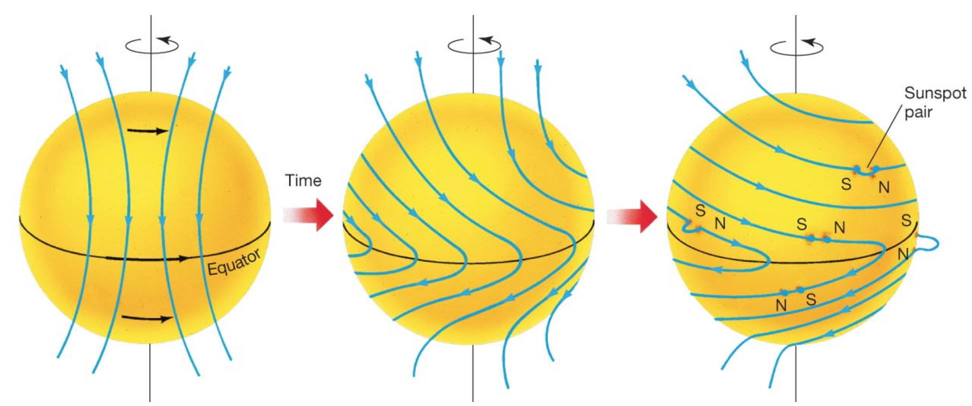
$$f(B) = \frac{1}{4}\left(1 + \mathrm{erf}\left(B^2 - B_{\mathrm{min}}^2\right)\right)\left(1 - \mathrm{erf}\left(B^2 - B_{\mathrm{max}}^2\right)\right)$$











Wilmot-Smith et al., ApJ 2006

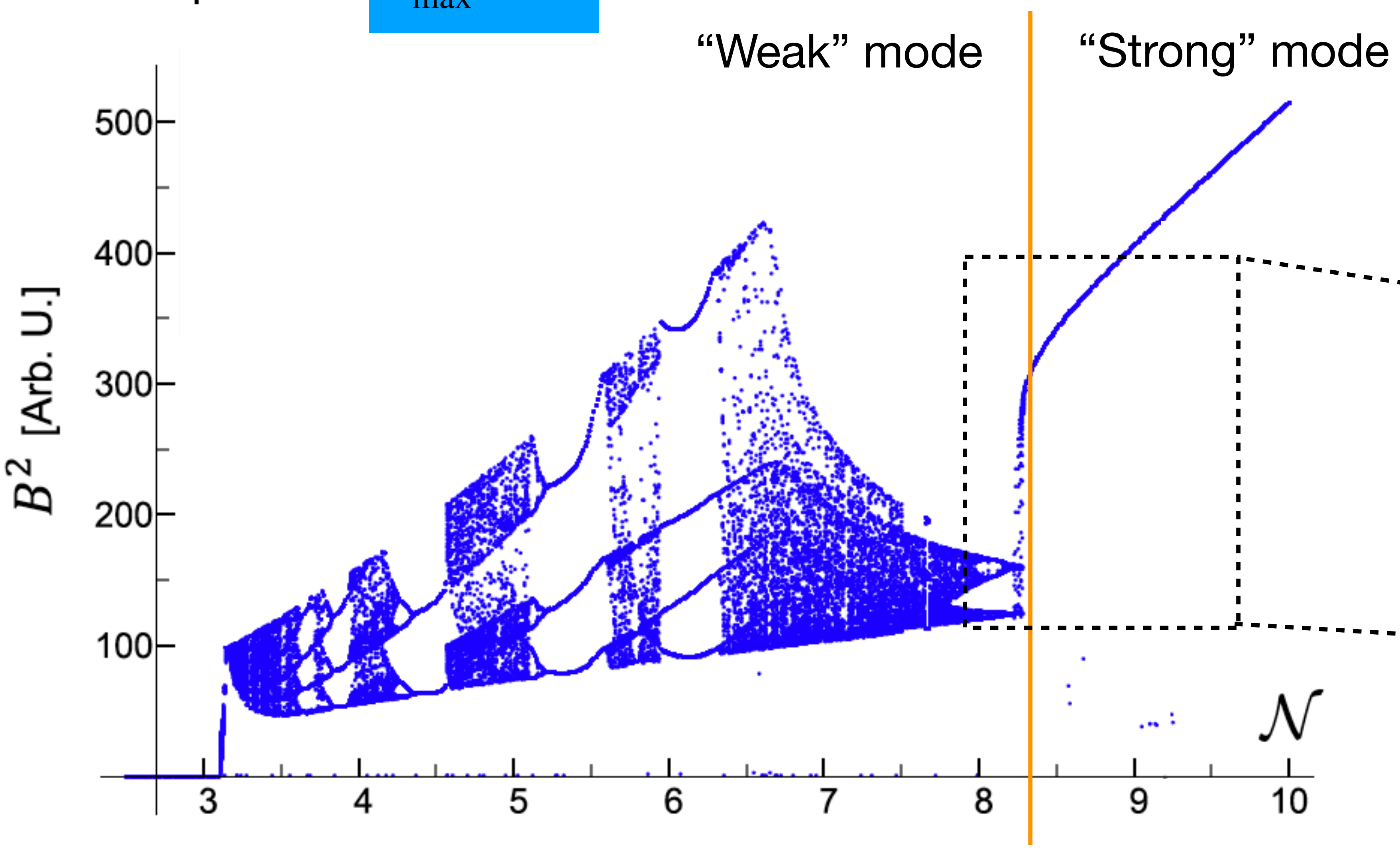
Ω effect: https://pages.uoregon.edu/jimbrau/astr122/Notes/Chapter16.html

The solar dynamo: a delay ODE model

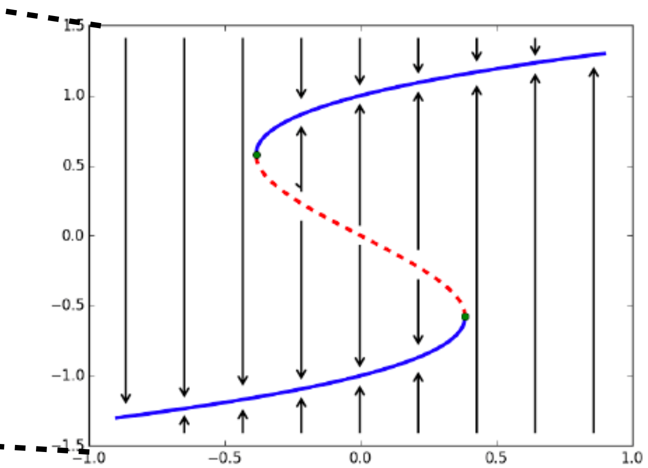
$$\left(\tau \frac{d}{dt} + 1\right)^2 B(t) = -\mathcal{N} \cdot f\left(B(t - T_0 - T_1)\right) \cdot B(t - T_0 - T_1)$$

$$\mathcal{N} = \frac{\alpha_0 \Omega_0 \tau^2}{L}$$

Field amplitude $B_{\max} = 10$



$\mathcal{N} \approx 8.4$



Typical bifurcation diagram showing *hysteresis / bi-stability*

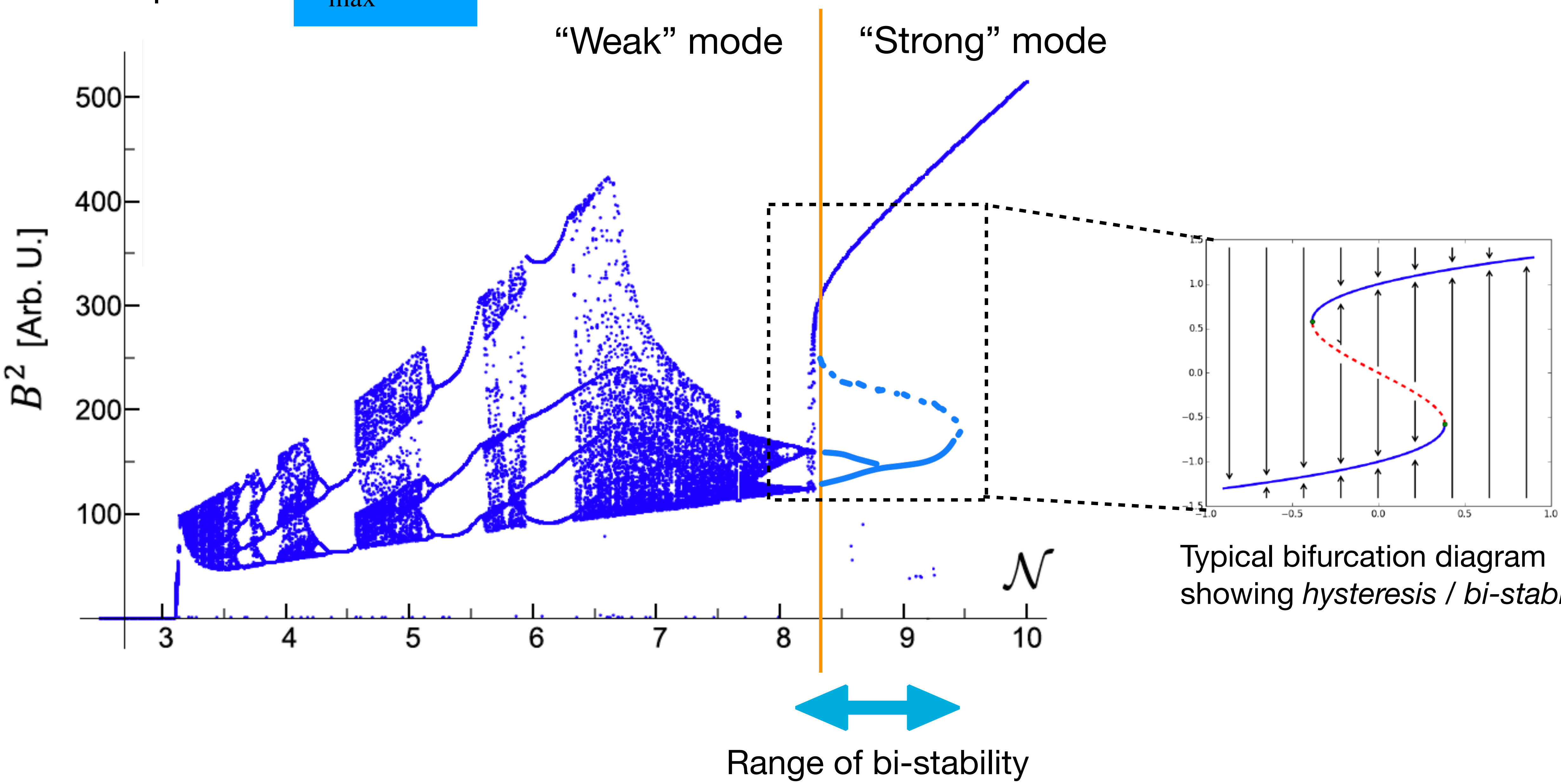
Albert, Ferriz-Mas, Gaia, Ulzega, ApJL 2021

The solar dynamo: a delay ODE model

$$\left(\tau\frac{d}{dt}+1\right)^2 B(t) = -\mathcal{N}\cdot f\left(B(t-T_0-T_1)\right)\cdot B(t-T_0-T_1)$$

$$\mathcal{N}=\frac{\alpha_0\Omega_0\tau^2}{L}$$

Field amplitude $B_{\max}=10$



Typical bifurcation diagram showing *hysteresis* / *bi-stability*

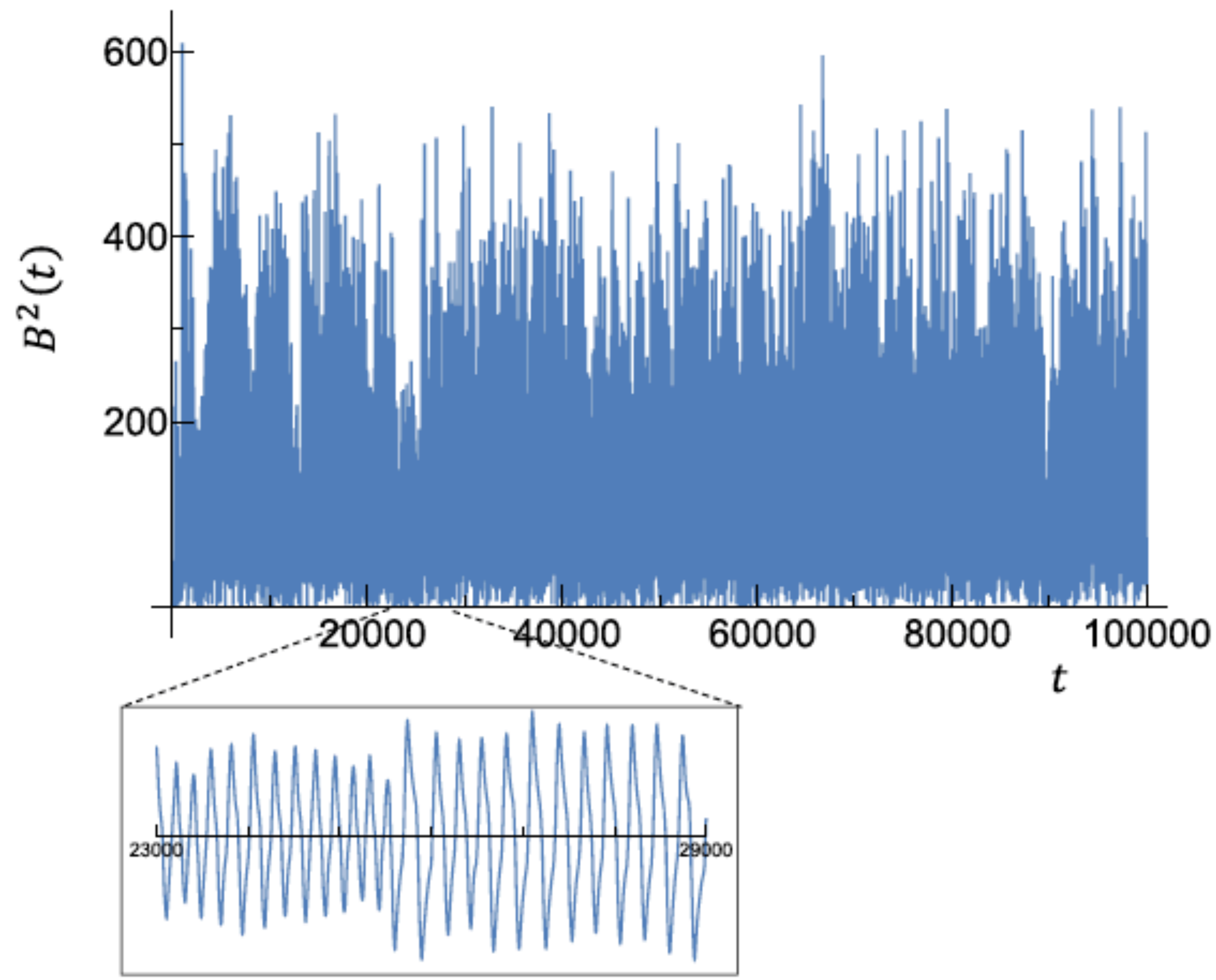
Albert, Ferriz-Mas, Gaia, Ulzega, ApJL 2021

The solar dynamo: a stochastic delay ODE model

$$\left(\tau\frac{d}{dt}+1\right)^2 B(t) = -\mathcal{N}\cdot f\left(B(t-T_0-T_1)\right)\cdot B(t-T_0-T_1)+\sqrt{\tau}B_{\max}\sigma\eta(t) \qquad \mathcal{N}=\frac{\alpha_0\Omega_0\tau^2}{L}$$

- **Noise** allows the system to switch between strong and weak modes

- **Stochastic resonance** could explain the periodic recurrence of **Grand Minima**



"Weak" mode
(Grand Minima)

"Strong" mode
(normal dynamo operation)

Albert, Ferriz-Mas, Gaia, Ulzega, ApJL 2021

And now?

- **Quantitative comparison** of **observed** and **simulated** features of solar magnetic activity

Bayesian inference

- **Quantitative comparison** of **observed** and **simulated** features of solar magnetic activity

THE BAYESIAN FRAMEWORK

Thomas Bayes
(1701 – 1761)
Statistician, philosopher,
Presbyterian minister

- **Knowledge (belief)** about model parameters $\boldsymbol{\theta}$ is expressed in the form of **probability distributions** conditioned on measured data $\mathbf{B}^{(\mathrm{obs})}$

BAYES' THEOREM

$$f\left(\boldsymbol{\theta} \mid \mathbf{B}^{(\mathrm{obs})}\right) \propto f\left(\mathbf{B}^{(\mathrm{obs})} \mid \boldsymbol{\theta}\right) f\left(\boldsymbol{\theta}\right)$$

Posterior distribution: probability of model parameters given measured data

Likelihood function: probability that the model produces data $\mathbf{B}^{(\mathrm{obs})}$ for given model parameters $\boldsymbol{\theta}$ (**probabilistic model** for data generating process)

Prior knowledge about parameters

What do we win?

learn something about the system

make reliable **probabilistic predictions**

Bayesian inference with ^{14}C-based data

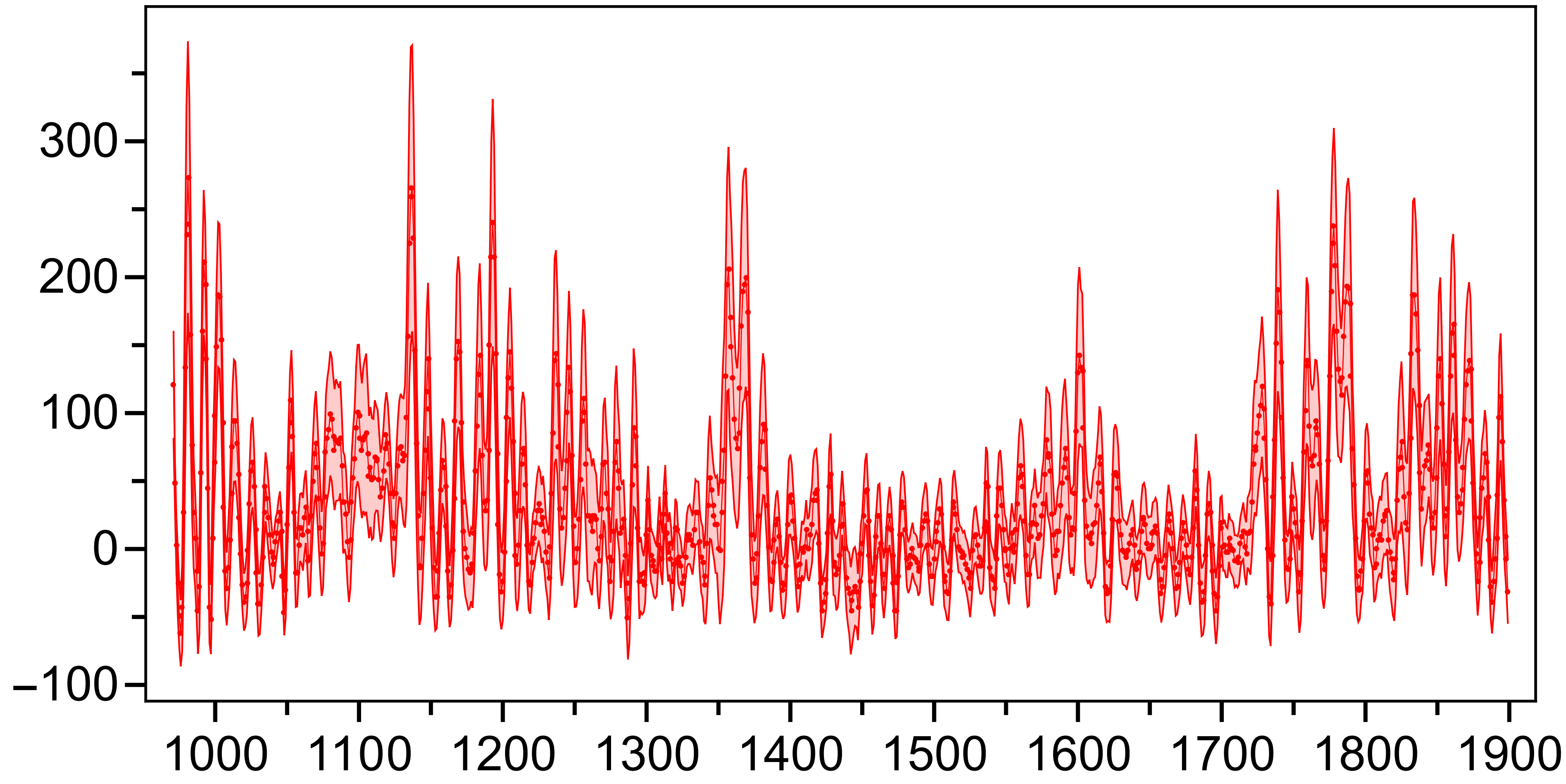
Model:

$$\left(\tau \frac{d}{dt} + 1\right)^2 B(t) = -\mathcal{N} \cdot f\left(B(t - T_0 - T_1)\right) \cdot B(t - T_0 - T_1) + \sqrt{\tau} B_{\max} \sigma \eta(t)$$

Parameters to be inferred: $\boldsymbol{\theta} = \left\{\tau, T, \mathcal{N}, \sigma, B_{\min}, B_{\max}\right\}$ with $T = T_0 + T_1$

Data:

Reconstructed SN *(Usoskin et al., A&A 2021)*



Approximate Bayesian Computation (ABC)

- **Approximate Bayesian Computation (ABC)** methods **bypass the prohibitively expensive evaluation of the likelihood** function $f\left(\mathbf{B}^{(\text{obs})} \mid \boldsymbol{\theta}\right)$

- ABC uses the model to **simulate** a large number (millions!) of realisations $\{\mathbf{B}_i\}$ for different parameter sets $\{\boldsymbol{\theta}_i\}$

- Simulated data are **compressed into a low-dimensional set of *summary statistics*** $\mathbf{s}_i = \mathcal{S}(\mathbf{B}_i)$ and **compared** with observations, $\mathbf{s}^{(\text{obs})} = \mathcal{S}(\mathbf{B}^{(\text{obs})})$

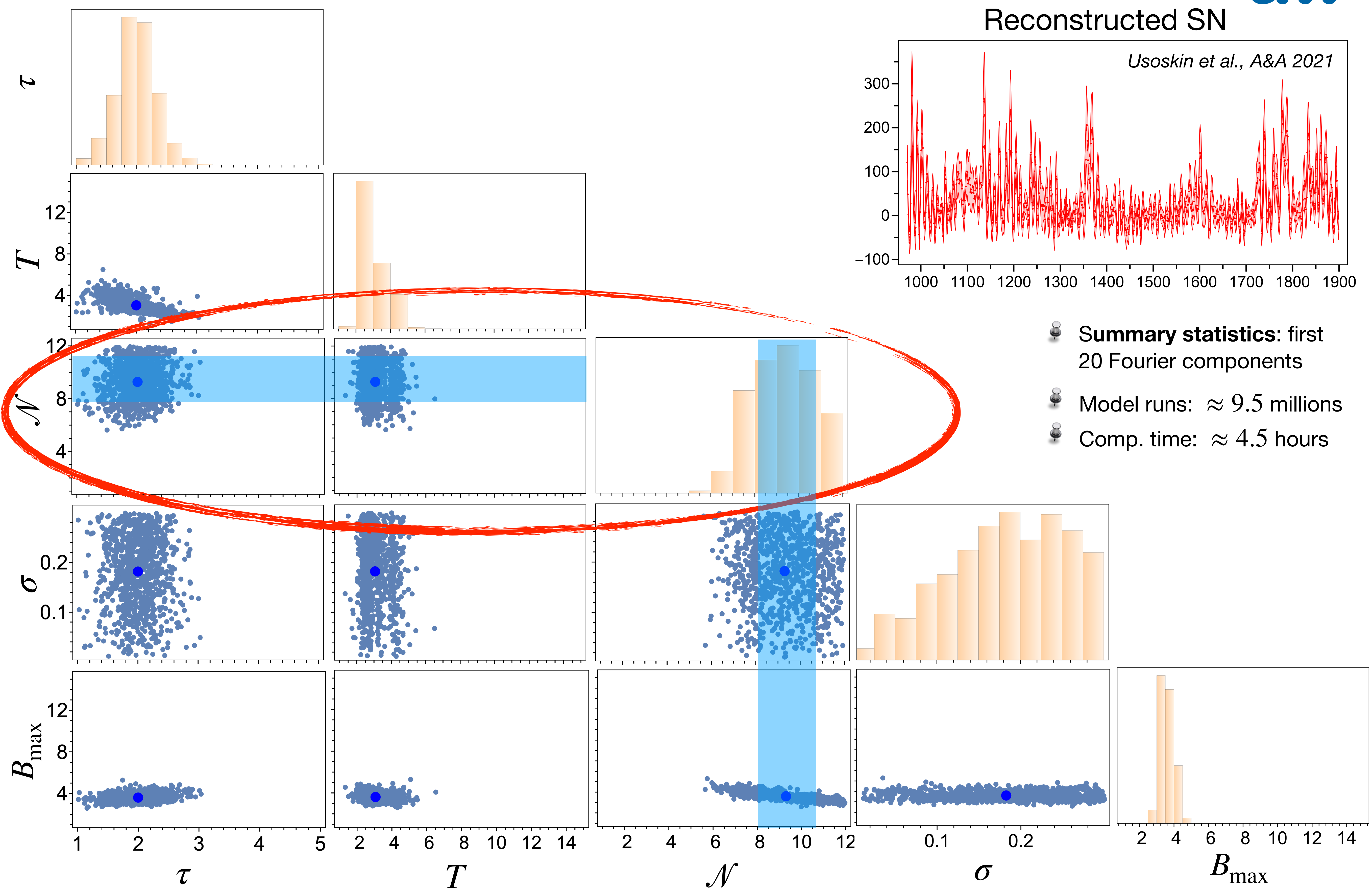
- Parameter sets are **accepted** with tolerance $\delta > 0$ if $\rho\left(\mathbf{s}_i, \mathbf{s}^{(\text{obs})}\right) \leq \delta$

  where the **distance** ρ quantifies the discrepancy between simulations and observations

ABC automatises what one would do manually:
simulate model outputs and compare them to observations!

Albert et al., Statistics and Computing 2015

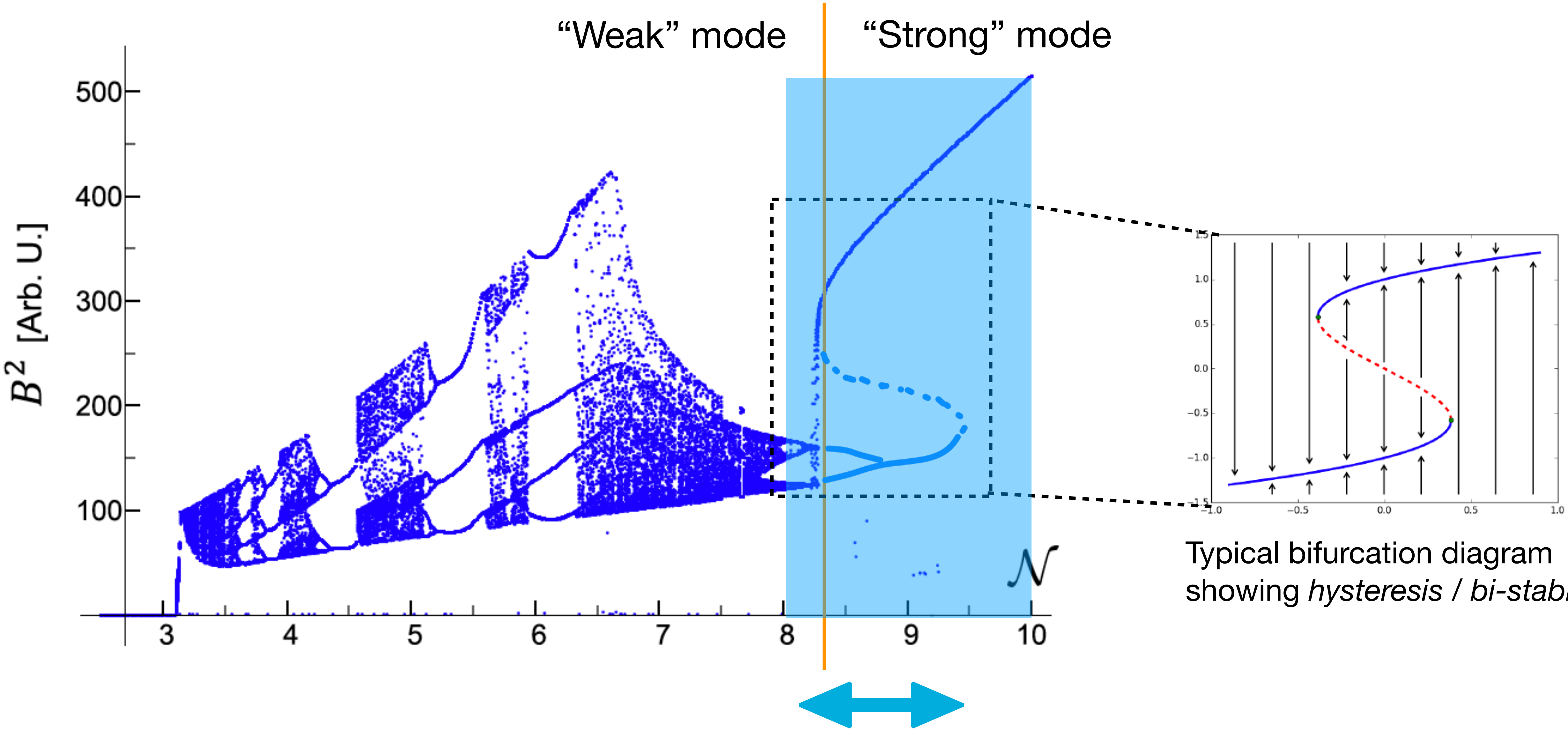
ABC (preliminary!) results



- **Summary statistics**: first 20 Fourier components
- Model runs: ≈ 9.5 millions
- Comp. time: ≈ 4.5 hours

ABC (preliminary!) results

Field amplitude



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