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Solar radiance variability in terms of the viral theorem

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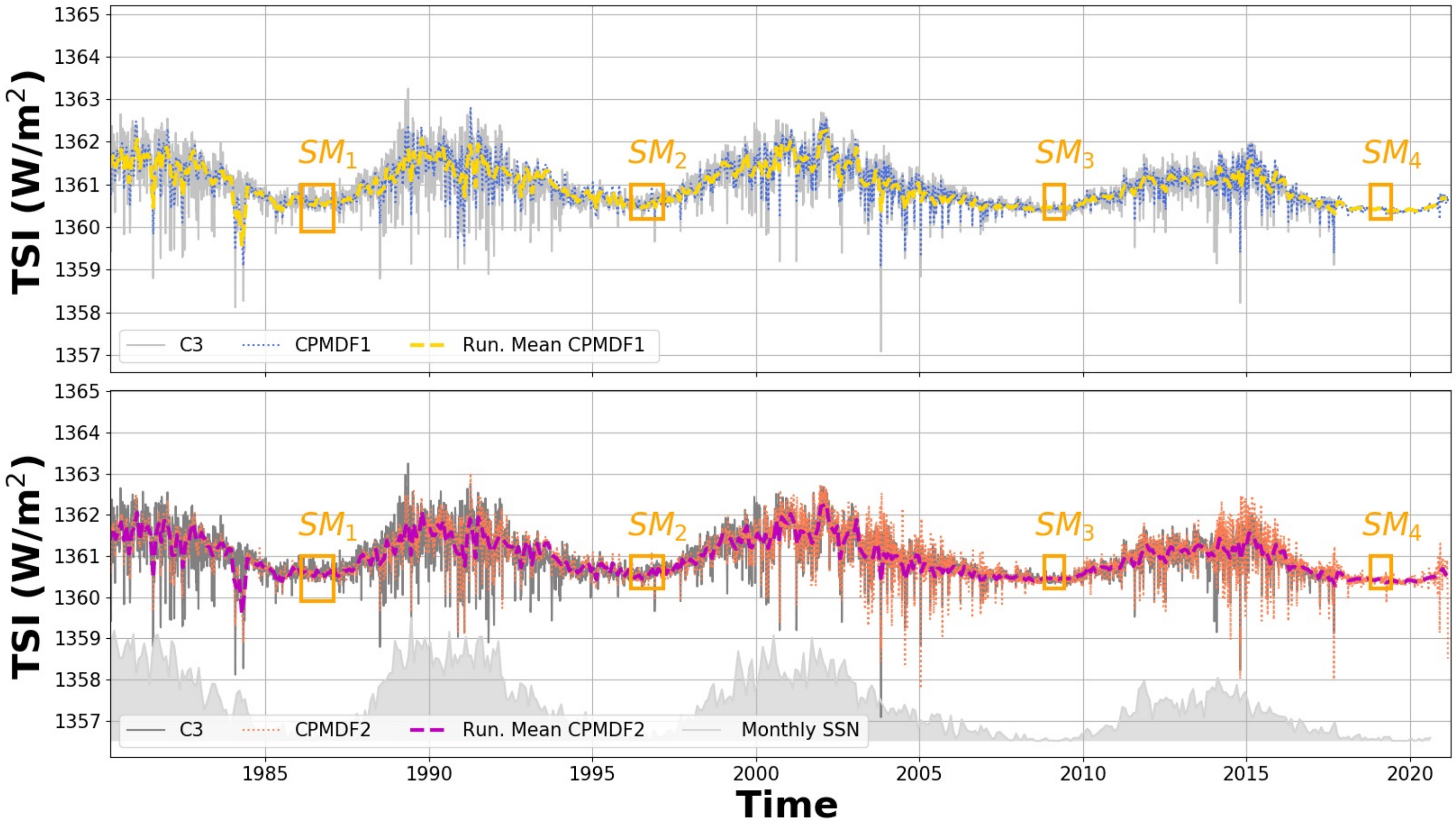
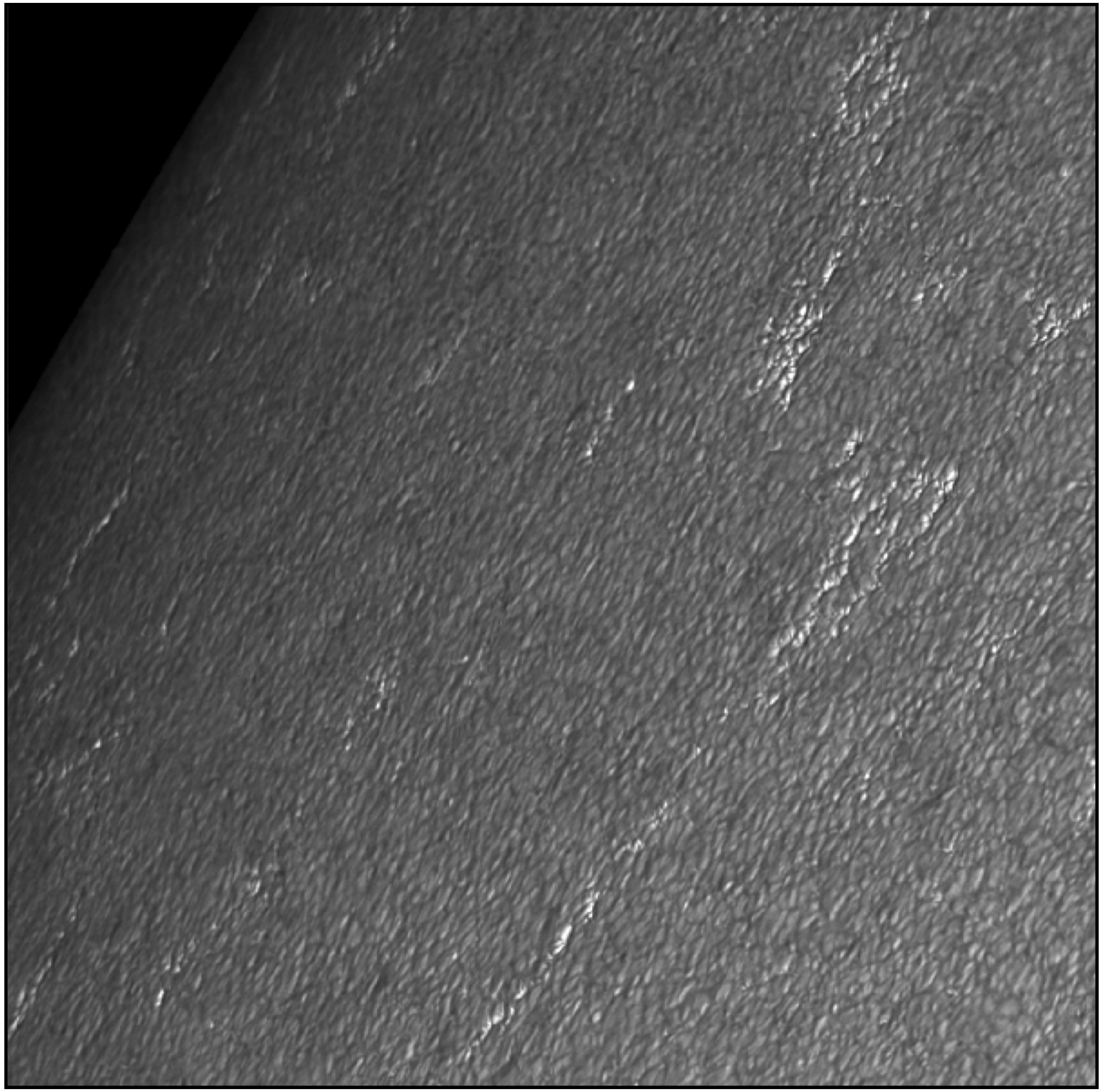
1. Solar radiance variability

The total solar irradiance ("solar constant") varies in phase with the solar cycle by $\approx 0.07\%$, peak-to-peak.



New Composite CPMDF1 and CPMDF2 without and with a wavelet filter, respectively.

From www.pmodwrc.ch

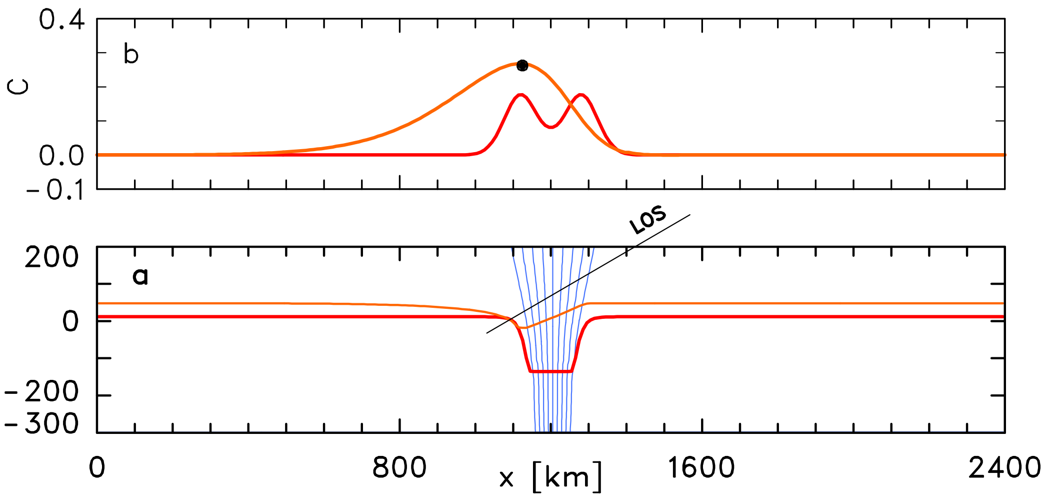


Facular region in the continuum at 487.5 nm. Field of view approximately $80'' \times 80''$.

From Hirzberger & Wiehr (2005), A&A 438, 1059

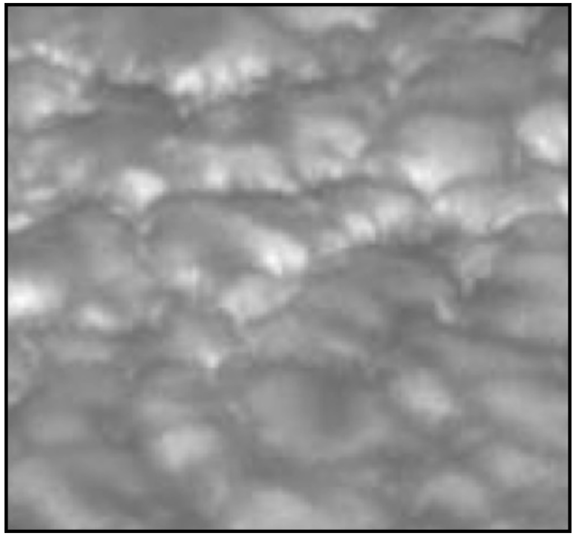
The cross-sectional area for excess radiative escape from faculae is much larger than the magnetic field concentration proper.

model



From Steiner, O. 2005, A&A 430, 691–700

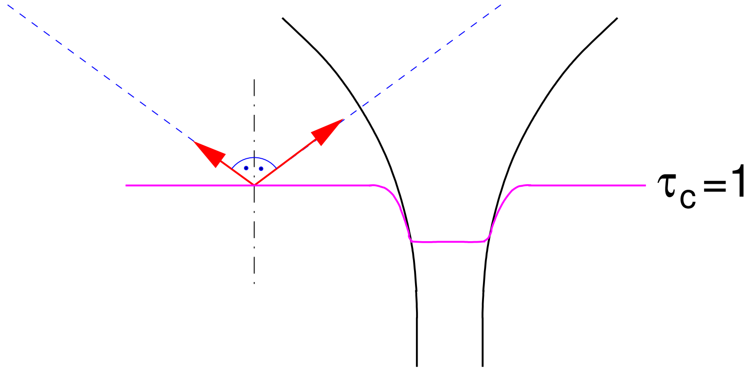
facular granule



From Lites, B. et al. 2004, Sol. Phys. 221, 65

Effect first described by *Caccin, B. & Severino, G. 1979, ApJ 232, 297–303*

- From a location at the solar surface and lateral to the flux sheet, a material parcel “sees” a *more transparent sky* in the direction to the flux sheet compared to a direction under equal zenith angle but away from it.



- Correspondingly, from a *wide area* surrounding the magnetic flux sheet or flux tube, radiation escapes more easily in the direction of the flux sheet/tube.

- A single flux sheet/tube impacts the radiative escape in a cross-sectional area (*“radiative cross section”*) that is much wider than the magnetic field concentration proper.

$\longrightarrow$ more on faculae

2. The virial theorem

The virial theorem is widely known as

$$2U + \Omega = 0 \ ,$$

where U is the total thermal energy and Ω is the gravitational binding energy of the star. Thus, any change $-\Delta\Omega$, e.g., by gravitational contraction of a star, leads to a change in the thermal energy of the star of

$$\Delta U = -\frac{1}{2}\Delta\Omega = \frac{1}{2}|\Delta\Omega| \ ,$$

leaving an energy excess of $-\Delta\Omega/2$, which must be lost from the star, normally in the form of radiation.

A *general form of the virial theorem*, including the magnetic field, is given by:

$$\boxed{\frac{1}{2}\frac{\mathrm{d}^2 J}{\mathrm{d}t^2} = 2K + \Omega + M + 3\int_{\mathcal{R}} P \mathrm{d}V + S\ ,}$$

Chandrasekhar & Fermi (1953)

$$J(t) = \int_{\mathcal{R}} \rho \mathbf{r}^2 \mathrm{d}V\ , \qquad M(t) = \int_{\mathcal{R}} \frac{\mathbf{B}^2}{2\mu_0} \mathrm{d}V\ ,$$

$$K(t) = \frac{1}{2}\int_{\mathcal{R}} \rho \mathbf{v}^2 \mathrm{d}V\ , \qquad 3\int_{\mathcal{R}} P \mathrm{d}V = 3(\gamma - 1) U(t)\ ,$$

$$\Omega(t) = \frac{1}{2}\int_{\mathcal{R}} \rho \Psi \mathrm{d}V\ , \qquad S(t) = -\oint_{\partial\mathcal{R}} P_{\mathrm{tot}} (\mathbf{r}\cdot\mathbf{n})\, \mathrm{d}S + \frac{1}{\mu_0}\oint_{\partial\mathcal{R}} (\mathbf{r}\cdot\mathbf{B})(\mathbf{B}\cdot\mathbf{n})\, \mathrm{d}\mathbf{S}\ .$$

where J is the moment of inertia, K the kinetic energy of mass motion, Ω the gravitational binding energy of the star with Ψ being the gravitational potential, M the magnetic energy, U the internal energy, and S a surface integral over the boundary $\partial\mathcal{R}$ of the region $\mathcal{R}$ occupied by the star.

3. Energy estimates

- The equivalent variation in thermal energy over a solar cycle due to the solar luminosity variation is $\int_{\text{cycle}} \delta L \mathrm{d}t \approx 0.5 \cdot \tau_{\text{cycle}} \cdot 0.001 \cdot L_{\odot}$ so that

  $\boxed{\Delta U_{\text{rad}} \approx 10^{32} \text{[J]}}$

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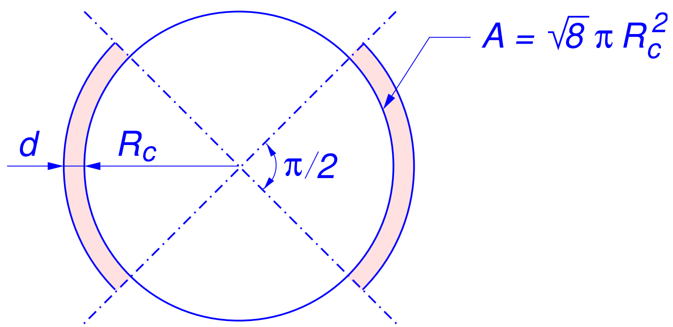
- One solar cycle generates a magnetic flux of $\Phi \approx 10^{16}$ Wb. If this flux resides in the overshoot layer in the form of flux sheet with a strength of 10 T, its total magnetic energy can be computed:



$$\frac{1}{2} R_c \pi d B = \Phi \quad \Rightarrow \quad d = \frac{2\Phi}{R\pi B} \approx 10^6 \text{[m]}$$

$$\boxed{E_{\mathrm{mag}} = \sqrt{8}\pi R_c^2 d \frac{B^2}{2\mu_0} \approx 10^{32} \text{[J]}}$$

- "Coincidence" of equal magnetic and thermal energy change (Schüssler, 1996)

The inertial virial: $\mathrm{d}^2J/\mathrm{d}t^2 \approx J/\tau_{\rm cyc}^2 \approx \int_{\mathcal{R}} \rho \mathbf{r}^2 \mathrm{d}V/\tau_{\rm cyc}^2. \quad \Rightarrow$

$$\boxed{\mathrm{d}^2J/\mathrm{d}t^2 \approx 10^{-3} \times \Delta U_{\rm rad}}$$

The kinetic virial: The kinetic energy of convective motion in the convection zone is $K_{\rm con} = (1/2)\int_{\mathcal{R}} \rho \mathbf{v}^{\mathbf{2}}_{\rm con} \mathrm{d}V = 7.8 \times 10^{31}$ [J] $= 0.16 \times \Delta U_{\rm rad}$. In the tachocline, where magnetic intensification presumably takes place, the available convective kinetic energy reduces to $K_{\rm con} \approx 10^{-4} \times \Delta U_{\rm rad}$. Also, because $F_{\rm conv} \propto v_{\rm conv}^2 \overset{!}{<} 10^{-3}$, convective motion cannot be tapped in sufficient amounts. Since turbulent, convective motion is unlike to produce coherent large scale magnetic flux ropes in the overshoot layer, we assume the variation of $K_{\rm con}$ with the solar cycle to be negligible, $\Delta K_{\rm con} = 0$.

The kinetic energy of rotation in the convection zone is

$K_{\mathrm{rot}} = (1/2)\int_{\mathcal{R}} \rho(\Omega(\theta,\mathbf{r})\times\mathbf{r})^2 \mathrm{d}V = 5.5\times 10^{34}\,[\mathrm{J}] = 110\times \Delta U_{\mathrm{rad}}$.

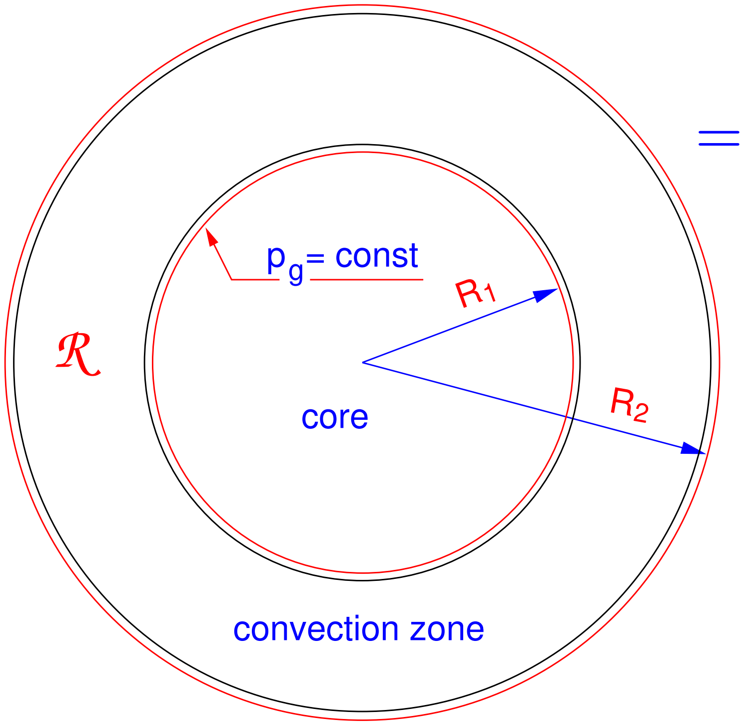
However, the difference to the kinetic energy of rigid rotation under the condition of equal rotational momentum, i.e., the available kinetic energy from differential rotation is $\Delta K_{\mathrm{rot}} = 1\times 10^{33}\,[\mathrm{J}] = 2\times \Delta U_{\mathrm{rad}}$.

The available kinetic energy from differential rotation in the tachocline alone amounts to $\Delta K_{\mathrm{rot}} = 1\times 8^{30}\,[\mathrm{J}] = 0.015\times U_{\mathrm{rad}}$.

Thus, differential rotation in the magnetic layer would have to be very efficiently replenished from layers atop for feeding the energy required for field intensification. In lack of such a mechanism we have

$$\boxed{K(t) = (1/2)\int_{\mathcal{R}} \rho\mathbf{v}^2 \mathrm{d}V = \mathrm{const}}$$

The region $\mathcal{R}$ encompasses the *convection zone*, where the lower boundary lies beneath the overshoot region in the radiative zone with $p_g = \mathrm{const}$ and $\mathbf{B} = 0$. At the upper boundary $P_g = 0$.



$$S(t) = -\oint_{\partial\mathcal{R}} P_{\mathrm{tot}}(\mathbf{r}\cdot\mathbf{n})\,\mathrm{d}S + \frac{1}{\mu_0}\oint_{\partial\mathcal{R}}(\mathbf{r}\cdot\mathbf{B})(\mathbf{B}\cdot\mathbf{n})\,\mathrm{d}\mathbf{S}$$

$$= \underbrace{4\pi R_1^3 p_g(R_1)}_{1.4\times 10^{40}\text{[J]}} + \underbrace{4\pi R_2^3 f \frac{\mathbf{B}^2(R_2)}{2\mu_0}}_{1.5\times 10^{28}\text{[J]}}$$

$$+ \underbrace{\frac{1}{\mu_0} 4\pi R_2^3 f B_\perp^2(R_2)}_{3\times 10^{28}\text{[J]}}$$

Considering the *variations* over a solar cycle, only *three virials* of the full virial equation remain significant:

$$\boxed{\delta\Omega + \delta M + 2\delta U = 0\ .}$$

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$$\Omega_f - \Omega_i + M_f - M_i + 2(U_f - U_i) = 0 \quad \text{virial equation}$$

$$\Omega_i + M_i + U_i = \Omega_f + M_f + U_f + R^{\nearrow} \quad \text{energy equation}$$

$$\Longrightarrow \boxed{R^{\nearrow} = \tfrac{1}{2}(\Omega_i - \Omega_f) + \tfrac{1}{2}(M_i - M_f)}$$

Energy estimates (cont.)

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$$\Longrightarrow \boxed{R^{\nearrow} = \tfrac{1}{2}(\Omega_i - \Omega_f) + \tfrac{1}{2}(M_i - M_f)}$$

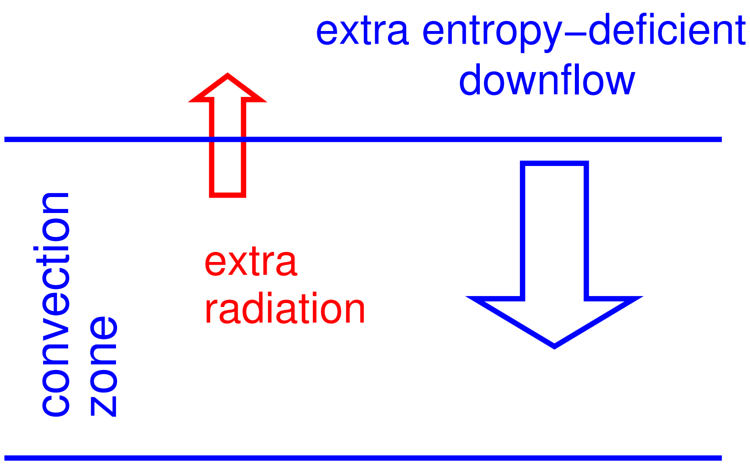
case 1: $M_i - M_f = \Delta M > 0$ and $\Delta\Omega = 0 \quad \Longrightarrow R^{\nearrow} = \dfrac{1}{2}|\Delta M|$

case 2: $\Omega_f - \Omega_i = -|\Delta M| \Rightarrow \Delta U = 0 \quad \Longrightarrow R^{\nearrow} = 0$

case 3: $\Delta M = 0 \Rightarrow U_f - U_i = -\tfrac{1}{2}(\Omega_f - \Omega_i) \quad \Longrightarrow R^{\nearrow} = \dfrac{1}{2}(\Omega_i - \Omega_f)$

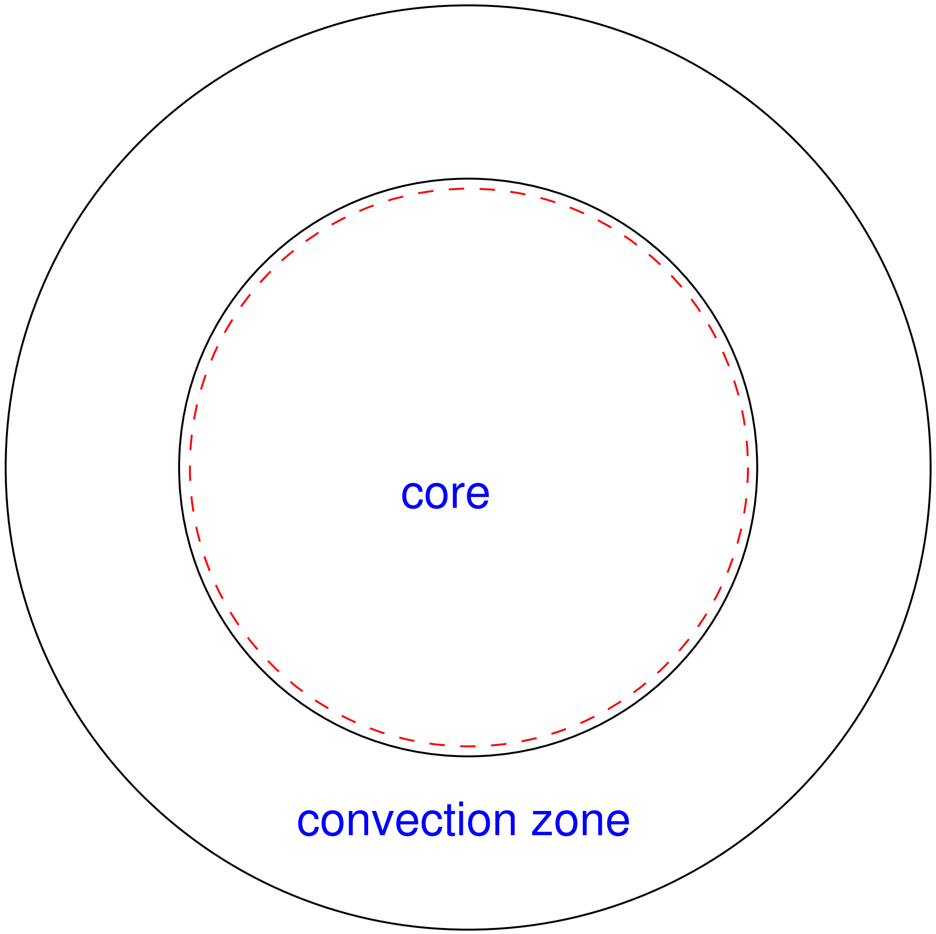
4. Entropy flow in the solar convection zone

We have a *downdraft of extra entropy deficient material* beneath magnetically active regions. Thus, the increase in emissivity from the solar surface is communicated to the deep convection zone on a hydrodynamical time scale.



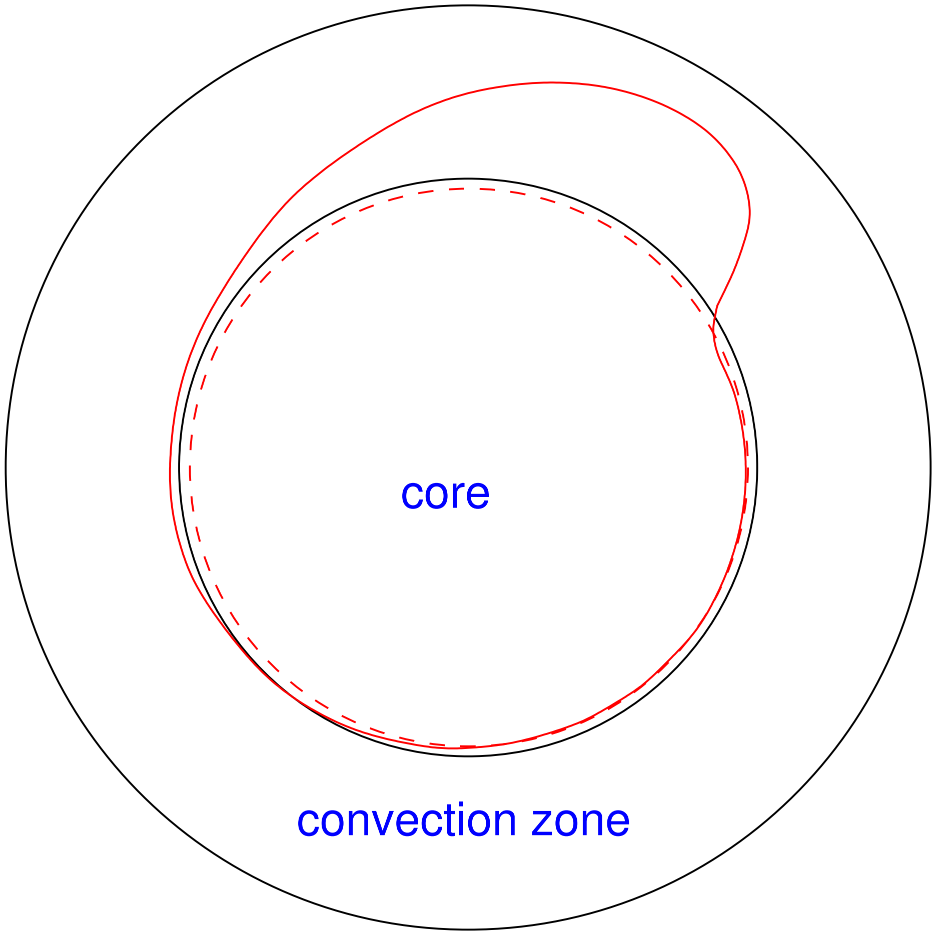
Spruit (2003) Sol. Phys. 213, 1-21 argues that this downdraft acts like a low pressure system on the convection zone that drives a quasi geostrophic flow, which ultimately causes the observed “torsional oscillation”.

Entropy flow in the solar convection zone (cont.)



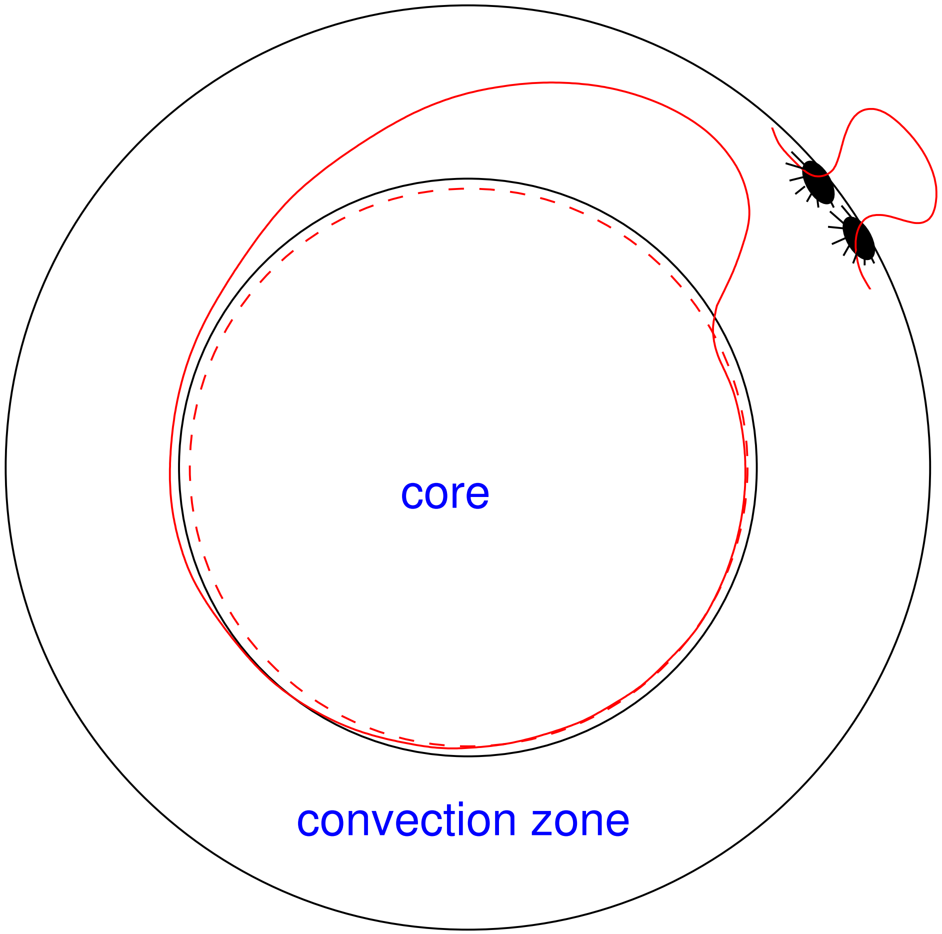
Magnetic flux tube at the bottom of the convection zone (dashed circle),

Entropy flow in the solar convection zone (cont.)



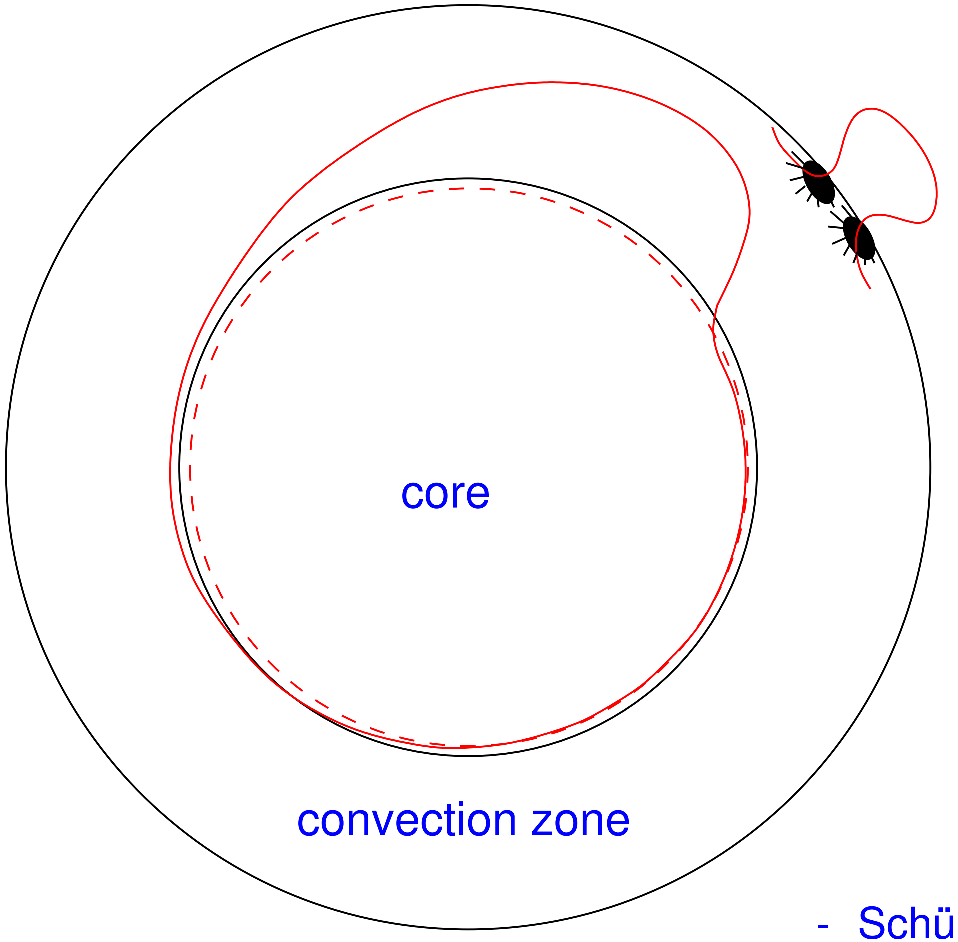
Magnetic flux tube at the bottom of the convection zone (dashed circle), rises through the convection zone to the surface (solid red circle),

Entropy flow in the solar convection zone (cont.)



Magnetic flux tube at the bottom of the convection zone (dashed circle), rises through the convection zone to the surface (solid red circle), where it forms a sunspot pair.

Entropy flow in the solar convection zone (cont.)

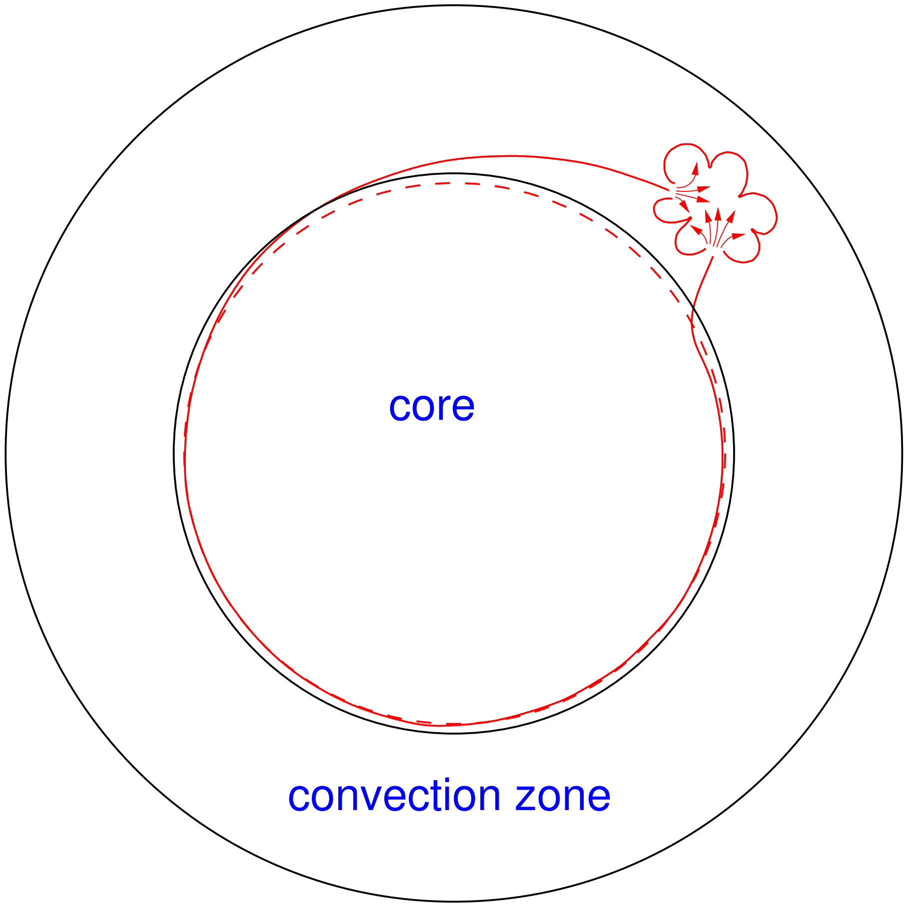


Magnetic flux tube at the bottom of the convection zone (dashed circle), rises through the convection zone to the surface (solid red circle), where it forms a sunspot pair.

The flux tube must have an *initial field strength of 10 T* in order to make it to the surface.

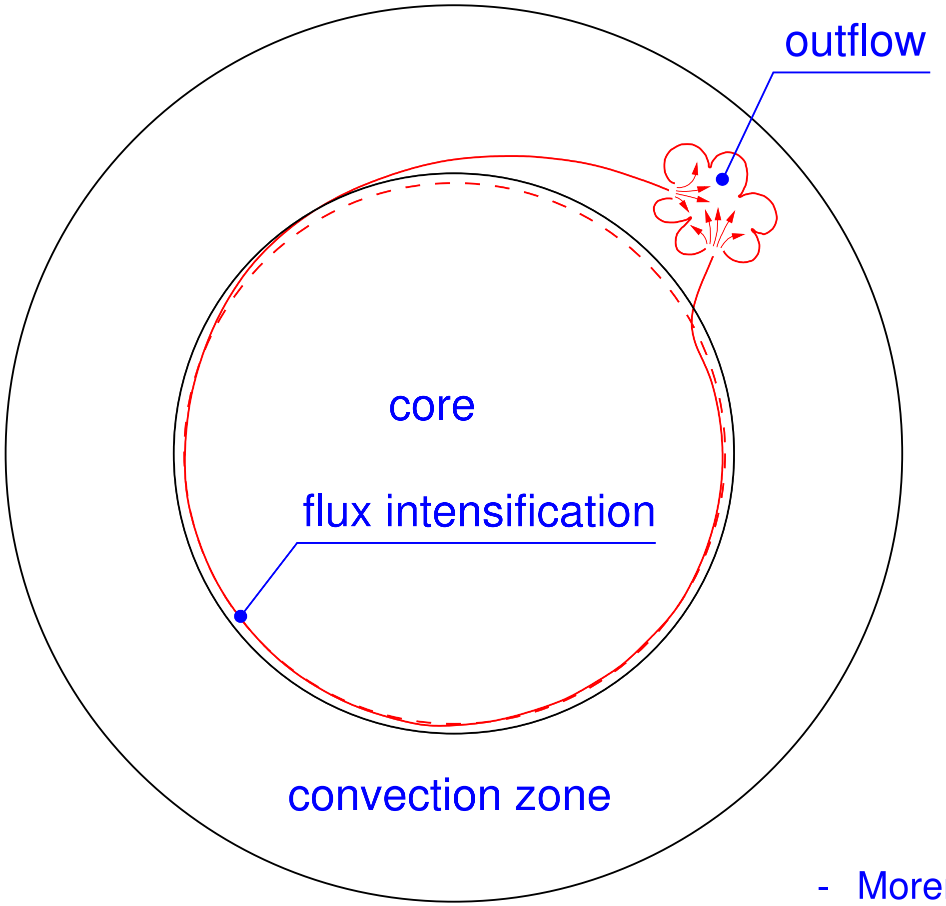
- Schüssler, Caligari, Ferriz-Mas, Moreno Insertis et al.
- Fisher, Fan, Longcope, Linton et al.
- D'Silva, Choudhuri et al.

Entropy flow in the solar convection zone (cont.)



If the *initial field strength is less than 10 T*, the flux tube gets out of pressure balance within the convection zone.

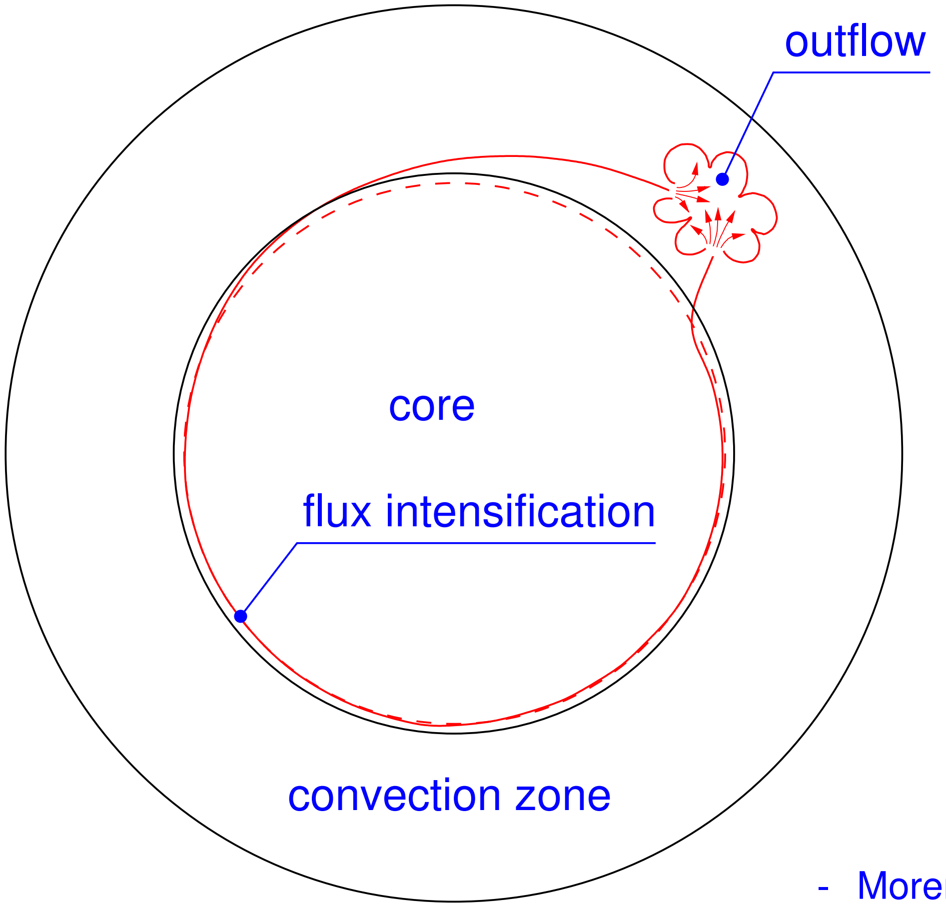
Entropy flow in the solar convection zone (cont.)



If the *initial field strength is less than 10 T*, the flux tube gets out of pressure balance within the convection zone. It “explodes” and an *outflow* ensues that leads to a *flux intensification* in the anchored part of the tube.

- Moreno-Insertis, Caligari, & Schüssler 1995, ApJ 452, 894
- Rempel & Schüssler 2001, ApJ 552, L171

Entropy flow in the solar convection zone (cont.)



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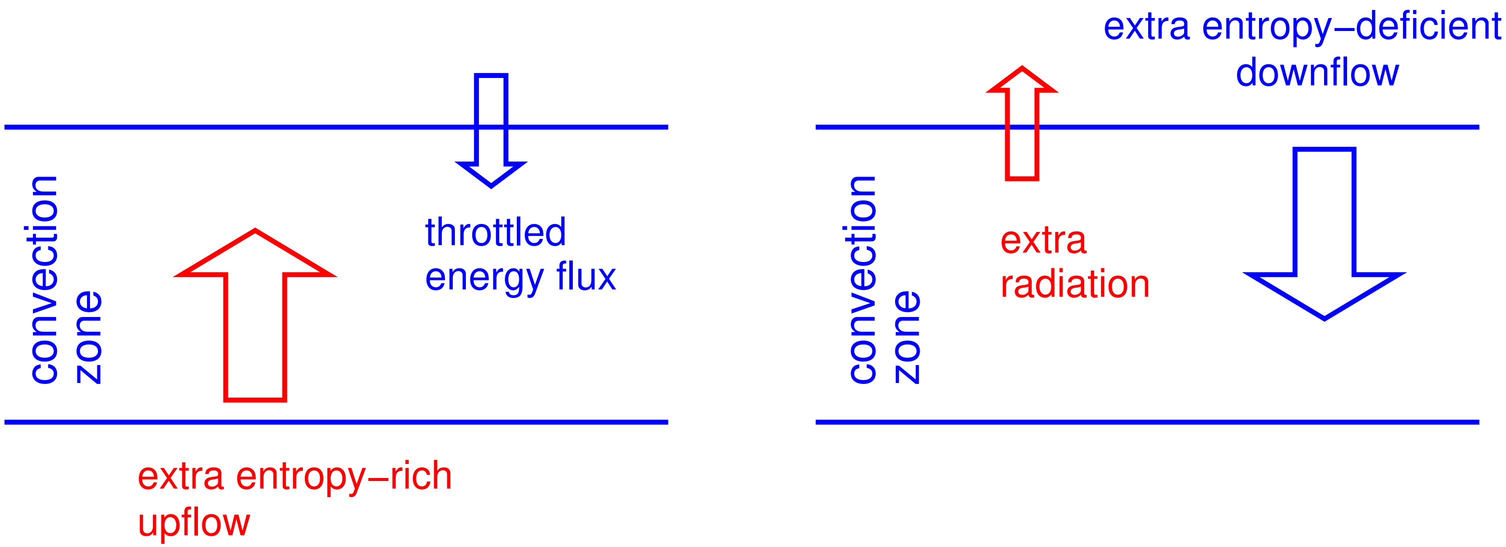
Note that the outflow *injects entropy rich material* to the convection zone.

Hence, we have an *extra entropy rich upflow* at times of the magnetic field intensification. This may possibly lead to a transient decrease in convective energy transport, $F_{\rm conv} \propto \delta$, because it reduces the superadiabaticity, δ.



These processes work on a *hydrodynamic time-scale* and involve the entire convection zone.

There is a *thermodynamic cycle* associated with the solar cycle.

5. Conclusions

- The solar cyclic oscillation of the total solar radiance is immediately caused by *surface magnetism*. However, there likely exists a *global thermal component* associated with it *comprising the entire convection zone.*
- This relationship can possibly be understood/interpreted in terms of the *virial theorem*.

Conclusions (cont.)

- *Observational consequences* may be
  - a variation of the superadiabaticity, δ in the convection zone dependent on the strength of the solar cycle. In particular, a sequence of strong cycles my lead to a reduction of δ over a longer time period;
  - the correlation between stellar luminosity variation and the total magnetic energy generated over a stellar cycle;
  - a possible correlation of luminosity with solar radius;
  - a readjustment of the convection zone at times of Maunder minima.

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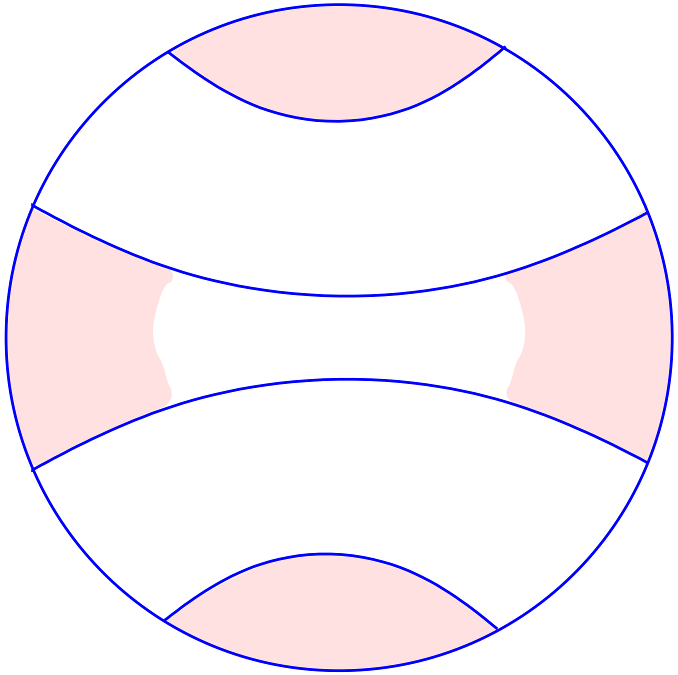
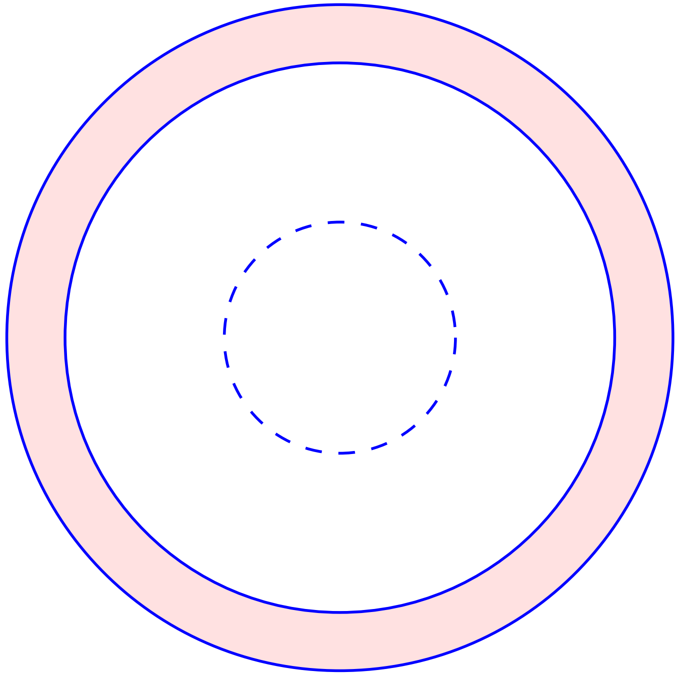
Facular regions as seen in the ecliptic plane and perpendicular to it



view in ecliptic plane



view perpendicular to ecliptic plane

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6. References

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